SUPPLEMENTARY MATERIAL: WHY ARE U.S. PARTIES SO POLARIZED? A "SATISFICING" DYNAMICAL MODEL

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1. Additional discussion of model assumptions. We discuss the empirical basis for the assumed ideology distribution of the voting population and key properties of a higher-dimensional extension of the model.

1.1. Population ideology distribution. In the main text, we assumed the public's ideology distribution to be unimodal and approximated it as Gaussian. Here we use empirical data to justify this assumption.

The American National Elections Studies (ANES) conducted yearly surveys from 1972 to 2012 asking people's self-identification on a 7-point scale between extremely liberal and extremely conservative. The self identification shows a unimodal distribution centered at "moderate." The data were downloaded in November 2015 from http://www.electionstudies.org/studypages/anes_timeseries_cdf/anes_timeseries_cdf.htm. The histogram for the U.S. public self-identified ideology is shown in Fig. S1. Here, the distribution remains peaked at the moderate position for all years, despite fluctuations, and the Gaussian distribution is a good approximation of this distribution.

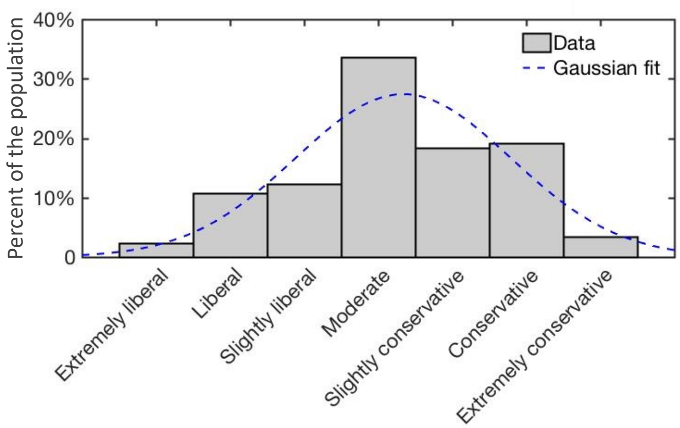

Fig. S1. *Distribution of self-reported ideology in the ANES data (1972–2012 aggregated) compared with a Gaussian fit. The R^2 of the Gaussian fit is 0.74.*

1.2. Dimensionality of the ideology space. In the main text, we are motivated by the reality that political idealogy in the U.S. is largely one-dimensional. The model can be generalized to higher dimensions by replacing the scalar variables x, μ_i, and σ_i with vectors. Figure S2 shows some example trajectories for the two-party case in a two-dimensional ideology space. In these simulations, for ease of numerical computation, we used discrete time steps and restricted party movements in each period to either vertical or horizontal directions, but the steady states reached are equivalent to those found by integrating the differential equations. The behavior of the system is analogous to that of the one-dimensional problem. For large values of the parameters σ_i, both parties converge to the center. For small-to-moderate σ_i, however, the parties stabilize at a finite separation on opposite sides of a circle (Figure S2).

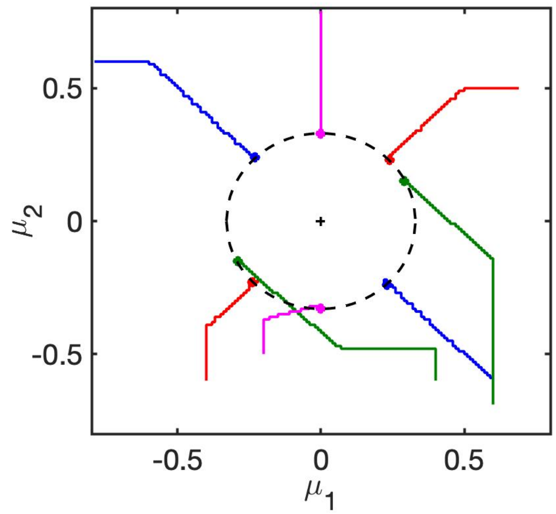

FIG. S2. *Example trajectories of the two-party system in a two-dimensional ideology space, where ideological positions are denoted by the coordinates* (μ_1, μ_2). *Solid circles denote the steady-state positions of the parties and the colored curves denote transient trajectories from various initial conditions. Curves of the same color indicate the two parties' trajectories from one set of initial conditions. In these simulations, the parameters are set to* $\sigma_0 = 1$ *and* $\sigma_{1,2} = 0.2$. *For simplicity, the simulation is restricted to parties moving parallel to the horizontal or vertical axis at each time step. The dashed circle is included to facilitate visualization of the steady-states positions.*

2. Data for the U.S. Democratic and Republican party positions. We use congressional voting records compiled with the Dynamic, Weighted, Nominal Three-Step Estimation (DW-NOMINATE) algorithm to empirically measure the party positions, μ, and the inclusiveness parameter, σ, for the Democratic and Republican parties. DW-NOMINATE is a multi-dimensional scaling method that first calculates a pairwise distance for every two members of Congress based on similarity in their roll call vote records. It then projects the resulting high-dimensional network of legislators to a low-dimensional space while preserving the pairwise distance relation as much as possible. The legislators' relative positions in this low-dimensional space are referred to as their ideology scores. This procedure was conducted for the House and Senate separately. The ideology scores of the two chambers were then combined into one dataset, which is the dataset used in our analyses.

We used the version of the House and Senate combined dataset that was last updated in 2015. The data was downloaded from https://legacy.voteview.com/dwnomin.htm in May 2016. The website has since been updated. The new link (as of September 2018) for data download is https://voteview.com/data. A copy of the dataset that the authors downloaded in May 2016 and used in this analysis can be found as a supplementary data file.

According to Poole and Rosenthal [S1], despite the underlying complexity, the roll call votes in the House and the Senate can be organized and explained by no more than two dimensions throughout American history. The first dimension, also called the dominant dimension, is commonly thought of as tapping into the left-right (or liberal–conservative) spectrum on economic matters. The second dimension picks up attitudes on cross-cutting and salient issues (e.g., slavery, civil rights, and social/lifestyle issues).

We use the DW-NOMINATE first dimension score to measure a legislator's ideological position. To estimate each party's position, μ_i, we use the mean position of a party's legislators. We also estimate each party's inclusiveness parameter, σ_i, as proportional to the standard deviation of the legislators' positions. The second dimension accounts for at most an additional 6%, and at the lowest point an additional 1%, of explanatory power in legislators' votes [S1].

3. Robustness checks. In this section, we provide evidence that the core conclusions of our paper do not change when using alternative datasets and slightly different model assumptions.

3.1. Validation for the satisficing function from independent data. For a robustness check of our findings, we employed an alternative estimate of the satisficing function parameter σ using a dataset that is independent of the DW-NOMINATE scores shown in the main text.

The data used is the American National Elections Studies (ANES) 1948-2012 time series dataset, as described in section 1 above. We used three variables from the survey: the self-reported liberal-conservative scale (VCF0803), the feeling thermometer for liberals (VCF0211), and the feeling thermometer for conservatives (VCF0212). Data on these variables are available for 18 years between 1972 and 2012, for 55,674 individuals in total.

The feeling thermometer questions ask participants to report their feeling towards a group on a 0-100 scale. Participants are told that 50–100 means they feel favorably towards the group, 0–50 means they do not feel favorably towards the group, and 50 means they do not feel particularly warm or cold.

We assume that thermometer ≥ 50 means satisfied with the liberal/conservative position. The self-reported liberal-conservative scale is 1 to 7, where 1 represents most liberal, and 7 most conservative. Figure S1 shows the aggregated distribution of the data from 1972 to 2012. Given the definition of the 7-point scale, we interpret the target group (liberal and conservative) as positioned at 1 and 7 of this scale, respectively. Then, we calculate the proportion of the population satisfied with the target group as a function of their distance to the group. We fit a Gaussian to this curve and get the parameter σ from the fit, which corresponds to the width of the satisficing function. Figure S3 shows, as an example, the behavior of the data in 2012. The two satisficing curves for conservatives and liberals collapse, and are well approximated by a Gaussian function.

We repeat the analysis for all 18 years of data available. Figure S4 plots the best fitting σ value to the ANES data of each year (horizontal axis) against the σ value estimated from the DW-NOMINATE data of the Congress that includes the same year. The Pearson correlation is 0.51 ($p = 0.03$). The σ value estimated from ANES correlates with distance between parties (i.e., party polarization) in the DW-NOMINATE data (Congress data) with Pearson correlation -0.58 ($p = 0.01$).

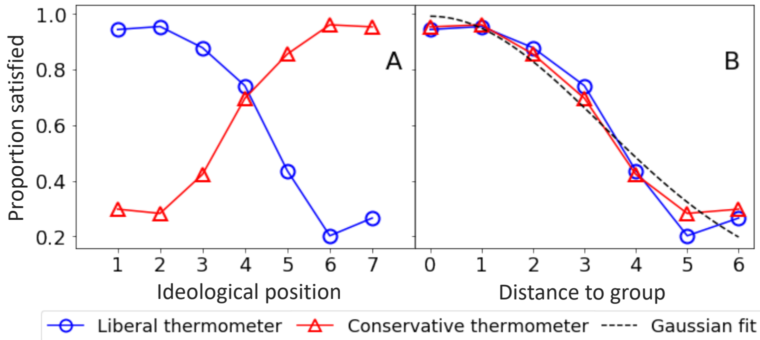

FIG. S3. *ANES thermometer data from 2012. We assume that thermometer* ≥ 50 *means satisfied. (A) Proportion of the population satisfied with liberals and conservatives by their ideological position, where 1 means extremely liberal and 7 means extremely conservative (B) Proportion satisfied as a function of the distance to the target groups (liberals or conservatives), and Gaussian fit. The two curves collapse and can be approximated by a Gaussian function. The best fitting Gaussian has a standard deviation of* 3.3.

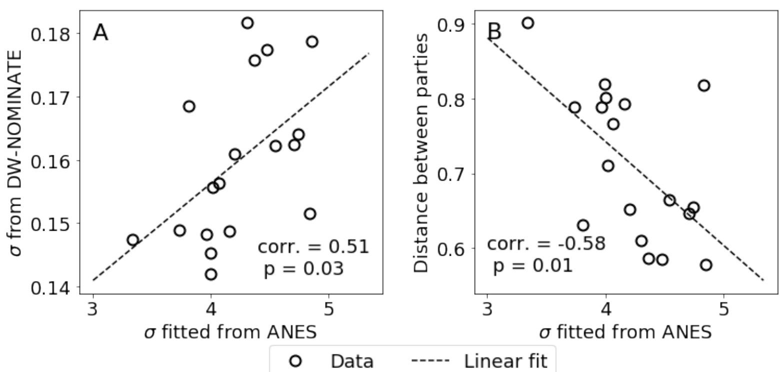

FIG. S4. *Comparing* σ *fitted from the ANES data and metrics computed from the DW-NOMINATE data. Left: the* σ *parameters inferred from the two sources are positively correlated, giving us some confidence in using the DW-NOMINATE standard deviation to approximate the "tolerance" of voters. Right: the parameter* σ *fitted from the ANES data is negatively correlated with the distance between parties (i.e., party polarization), which supports the observation made in the main text using* σ *derived from the DW-NOMINATE data.*

This analysis serves as a robustness check for the negative relationship between party separation (i.e., political polarization) and the narrowing of the satisficing function (i.e., increasing intra-party ideological homogeneity or reducing inclusiveness).

3.2. Polarization in alternative metrics of party position. The DW-NOMINATE score has been criticized at times for its computational complexity and for the difficulty of interpreting its axes. Here, we present a simpler and easier-to-understand metric and show that a negative relationship between polarization and inclusiveness is still observed.

We use data from the U.S. House of Representatives' roll call records for the period 1941–2015. The data were downloaded from https://www.govtrack.us/data/ in

March 2016.

We calculate two scores for each representative. For each representative in each Congress, we calculate a Republican alignment score (S_R). This score measures the proportion of times she or he voted in agreement with the Republican majority position. For example, if 60% of Republican representatives supported a bill, then a representative voting in favor of that bill would be counted as voting in agreement with the Republican majority. More explicitly,

$$S_R = \frac{N_R}{N}, \tag{3.1}$$

where S_R is the Republican alignment score, N_R is the number of times voting with the Republican majority, and N is the total number of votes. Similarly, we also calculate a Democratic alignment score for each representative,

$$S_D = \frac{N_D}{N}, \tag{3.2}$$

where N_D is the number of times a representative votes with the Democratic majority.

Finally, we combine the two scores into one alternative metric:

$$S = \frac{1}{2}(S_R - S_D). \tag{3.3}$$

Results of this analysis are shown in Fig. S5. The polarization trend is observable even using this simple metric (shown in the second column of Fig. S5). The negative relationship between the interparty distance (i.e., party polarization) and party inclusiveness is still present. The Pearson correlation in the DW-NOMINATE data is -0.77 ($p = 2 \times 10^{-16}$), and the Pearson correlation in the alternative metric is -0.59 ($p = 2 \times 10^{-8}$). Note that the Republican and Democratic scores (shown in the third and fourth columns of Fig. S5) exhibit asymmetry due to a number of unanimous votes.

4. Alternative model that maximizes vote share. In Eq. (3.3) of the main text, we assume that each party acts to maximize its number of votes. But in many cases parties may seek instead to maximize their vote *share*. We analyze this alternative model and find that similar qualitative conclusions hold; specifically, lower inclusiveness is related to greater party polarization. In the main text we chose to present the vote-maximizing model because it requires fewer assumptions and parameters.

In the alternative model for elections among parties 1 through n, Equation (3.3) is modified to

$$\frac{d\mu_i}{dt} = k\frac{\partial}{\partial \mu_i}\left(\frac{V_i}{\sum_{j=1}^{n} V_j}\right), \tag{4.1}$$

where n is the number of parties. Similar expressions for p can be derived for multiparty systems. For example, in a three-party system,

$$p_1 = \underbrace{s_1(1-s_2)(1-s_3)}_{\text{term 1}} + \frac{1}{2}\underbrace{(s_1 s_2(1-s_3) + s_1 s_3(1-s_2))}_{\text{term 2}} + \frac{1}{3}\underbrace{s_1 s_2 s_3}_{\text{term 3}}, \tag{4.2}$$

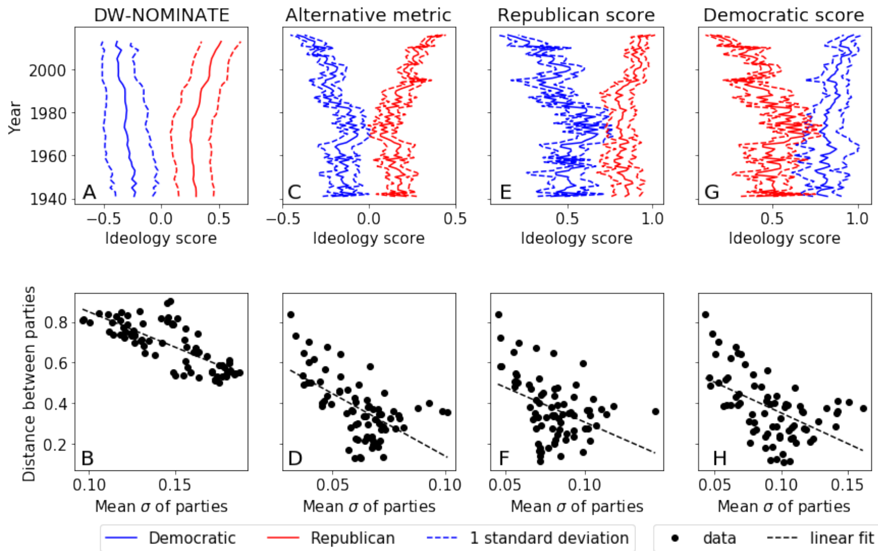

FIG. S5. *Party polarization trends from alternative metrics. Top row: time history of party positions and standard deviations measured by the various metrics. Bottom row: distance between parties (i.e., party polarization) vs. party inclusiveness based on various metrics. (A, B) DW-NOMINATE data (same as used in Fig. 4 of the main text). (C, D) Alternative metric S. (E, F) Republican alignment score S_R. (G, H) Democratic alignment score S_D.*

where the functions s_1, s_2, and s_3 are the same satisficing functions as described in the main text. Term 1 gives the expected proportion of voters satisfied with party 1 only. Term 2 gives the proportion satisfied with party 1 and exactly one other party. Term 3 gives the proportion satisfied with all three parties.

In the case where exactly two parties compete, Eq. (4.1) predicts that both parties will converge to the median voter's position, recovering a classic result in political science [S4]. However, this strategy is sensitive to the presence of even very small third parties. In the multi-party version of Eq. (3.3), with a minor third party (characterized by a small σ_3 parameter), the outcome changes: the two major parties diverge from the median and stabilize at a finite distance from each other. Intuitively, because voters who are unsatisfied with either party choose to abstain, in a strictly two-party system, the median voter's position does not maximize the number of votes for either party, although it does maximize the vote share. It is only with the threat of additional parties capturing lost votes that convergence to the median ceases to be optimal. This prediction is in agreement with previous research that finds that third-party candidates can shape election outcomes [S3]. We also perform analyses similar to those shown in Fig. 4 using a four-party version of Eq. (3.3), consisting of two major parties and two minor parties, and find qualitatively similar results (an example is shown in Fig. S6).

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[S1] K.T. Poole, H.L. Rosenthal, *Ideology and Congress* (Transaction Publishers, 2011).
[S2] H. Hermsen, A. Verbeek, Equilibria in multi-party systems, *Public Choice*, **73**, 147 (1992).

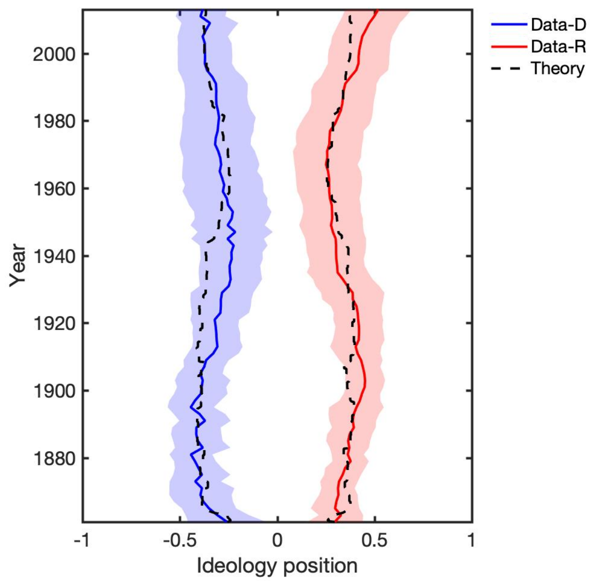

Fig. S6. *The comparison of party position over time predicted by the alternative model for maximizing vote share with empirical data. In the alternative model, we assume two major parties (shown), and two minor parties on either side of the major parties (not shown).*

[S3] Lee, D. J. Anticipating entry: Major party positioning and third party threat, *Polit. Res. Quart.*, **65**, 138–150 (2012).

[S4] Downs, A. An economic theory of political action in a democracy, *J. Polit. Econ.*, **65** (1957), pp. 135150.